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Some Results on Neutrosophic I-Statistically Convergence

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Abstract:

This paper introduces the neutrosophic I-statistical convergent difference sequence spaces defined through a modulus function. Additionally, we establish new topological spaces and examine various topological properties within these neutrosophic I-statistical convergent difference sequence spaces.

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1. INTRODUCTION

In the 1990s, Florentin Smarandache introduced neutrosophy as a response to challenges in addressing uncertainty. Neutrosophy seeks to examine and describe systems, phenomena, and concepts that encompass ambiguity, incompleteness, and indeterminacy, offering a more comprehensive perspective on these elements.

The Neutrosophic set [1,2], a mathematical framework extending fuzzy sets [3] and classic crisp sets, is central to Neutrosophy. A neutrosophic set's membership consists of three elements: truth, falsity, and indeterminacy. This structure finds application in diverse fields, such as decision-making, artificial intelligence, medical diagnosis, image processing, and pattern recognition. By accounting for the relationships among truth, falsity, and indeterminacy, neutrosophic sets provide a more adaptable and comprehensive approach to modeling real-world uncertainties, making them a valuable tool for handling complex and contradictory data. See [4] for other applications.

Bera and Mahapatra [4] first defined the neutrosophic soft linear space, and Bera and Mahapatra further explored neutrosophic soft norm linear spaces, metrics, convexity, and Cauchy sequences [6]. This paper aims to extend the intuitionistic fuzzy normed space framework to neutrosophic normed spaces, studying the properties of Cauchy sequences within this structure to uncover notable results.

The concept of statistical convergence was first introduced by Fast [7] and subsequently explored from the perspective of sequence spaces by numerous authors. The study of statistical convergence in double sequences was initiated independently by Mursaleen and Edely [8], and Tripathy [9]. Das *et al*. [10] extended this to$I$-convergenceof double sequences within metric spaces, analyzing several properties. Additional insights on statistical, I-convergence and I-statistical convergence can be found in the works of Dündar and Altay [11], Gürdal and Huban [12], Gürdal and Şahiner [13], Huban and Gürdal [14], Savaş and Gürdal [15, 16] and Savaş and Das [17].

Kirişci and Şimşek [18] further explored statistical convergence in neutrosophic normed spaces. Given that neutrosophic normed spaces generalize intuitionistic fuzzy normed spaces, statistical convergence within this framework presents a significant research interest. This motivated our focus on I-statistical convergence in neutrosophic normed spaces. For further details on ideal and statistical convergence, see [19, 20].

Kizmaz [21] developed the concept of difference sequence spaces by studying the difference sequence spaces $X = l_\infty(\Delta), c(\Delta), c_0(\Delta)$.Some novel sequence spaces were introduced by several authors [22-25].

In this study, we present the neutrosophic I-statistical convergent difference sequence spaces $I_{st(\Delta)}^{(X)}(f)$and $I_{st(\Delta)}^{0(X)}(f)$ defined by modulus function and investigate some of its topological properties.

2. MATERIALS AND METHODS

As seen below, Kizmaz [21] defines the difference sequence spaces using the difference matrix.

$$X(\Delta) = \{\zeta = \zeta_n : \Delta\zeta \in X\}$$

for $X = c, l_\infty, c_0$, where $\Delta\zeta_n = \zeta_n - \zeta_{n+1}$ and Δ shows the difference matrix $\Delta = (\Delta_{nm})$ defined by

$$\Delta_{nm} = \begin{cases} (-1)^{n-m}, & \text{if } n \leq m \leq n+1 \\ 0, & \text{if } 0 \leq m < n \end{cases} \tag{2.1}$$

Definition 2.1. [26] A function $f: [0,\infty) \to [0,\infty)$ is called a modulus function if the following conditions are met,

(a) $f(\zeta) = 0 \Leftrightarrow \zeta = 0$,

(b) $f(\zeta_1 + \zeta_2) \leq f(\zeta) + f(\zeta_2)$,

(c) f is non-decreasing, and

(d) f is continuous from the right at zero.

Since $|f(\zeta_1) - f(\zeta_2)| \leq f(|\zeta_1 - \zeta_2|)$, condition (d) implies that f is continuous on $\mathbb{R}^+ \cup \{0\}$.

Moreover, from condition (b) we have $f(n\zeta) \leq nf(\zeta), \forall n \in \mathbb{N}$, and so $f(\zeta) = f\left(n\zeta\left(\frac{1}{n}\right)\right)$. Hence $\frac{1}{n}f(\zeta) \leq f\left(\frac{\zeta}{n}\right) \forall n \in \mathbb{N}$.

It is possible for the modulus function to be either bounded or unbounded. Consider the following example: $f(\zeta) = \frac{\zeta}{1+\zeta}$, then $f(\zeta)$ is bounded. If $f(\zeta) = \zeta d, 0 < d < 1$, then the modulus function $f(\zeta)$ is unbounded.

Definition 2.2. A binary operation $\star$ on $[0,1]$ is referred to as CTN if (a) $\star$ is associative, commutative and continuous, (b) $\mu = \mu \star 1$ for any $\mu \in [0,1]$ and (c) for each $\mu_1, \mu_2, \mu_3, \mu_4 \in [0,1]$, if $\mu_3 \geq \mu_1$ and $\mu_4 \geq \mu_2$ then $\mu_3 \star \mu_4 \geq \mu_1 \star \mu_2$.

A binary operation $\circ$ on $[0,1]$ is referred to as CTCN if (a) $\circ$ is associative, commutative and continuous, (b) $\mu = \mu \circ 0$ for any $\mu \in [0,1]$ and (c) for each $\mu_1, \mu_2, \mu_3, \mu_4 \in [0,1]$, if $\mu_3 \geq v_1$ and $\mu_4 \geq \mu_2$ then $\mu_3 \circ \mu_4 \geq \mu_1 \circ \mu_2$.

Definition 2.3. Let $\mathcal{Y} \neq \phi$ and $X \subset \mathcal{Y}$. Then,

$$X_{NS} = \{< \zeta, \Pi(\zeta), \Xi(\zeta), \Phi(\zeta) >: \zeta \in \mathcal{Y}\}$$

where $\Pi(\zeta), \Xi(\zeta), \Phi(\zeta): X \to [0,1], \Pi(\zeta) =$ Truth, $\Xi(\zeta) =$ Indeterminacy, and $\Phi(\zeta) =$ Falsehood respectively.

$0 \leq \Pi(\zeta) + \Xi(\zeta) + \Phi(\zeta) \leq 3.$

The components of neutrosophic are $\Pi(\zeta), \Xi(\zeta)$ and $\Phi(\zeta)$ independent of each other.

Definition 2.4. Assume $\mathcal{Y}$ is a real vector space, $\star$ and $\diamond$ are CTN and CTCN, respectively, and $X = \{< \zeta, \Pi(\zeta), \Xi(\zeta), \Phi(\zeta) >: \zeta \in X\}$ be a neutrosophic set s.t $X: \mathcal{Y} \times (0, \infty) \to [0,1]$. The four-tuple $(\mathcal{Y}, X, \star, \diamond)$ is called a neutrosophic normed space (NNS) if the subsequent terms holds; $\forall \zeta, y \in X$ and $s, r > 0$

(i) $0 \leq \Pi(\zeta, s) \leq 1, 0 \leq \Xi(\zeta, s) \leq 1, 0 \leq \Phi(\zeta, s) \leq 1, s \in R^+$,

(ii) $\Pi(\zeta, s) + \Xi(\zeta, s) + \Phi(\zeta, s) \leq 3$, for $s \in R^+$,

(iii) $\Pi(\zeta, s) = 1$ iff $\zeta = 0$

(iv) $\Pi(\lambda\zeta, s) = \Pi\left(\zeta, \frac{s}{|\lambda|}\right)$,

(v) $(\Pi(\zeta, s) \star \Pi(y, r) \leq \Pi(\zeta + y, s + r)$,

(vi) $\Pi(\zeta, \star)$ is continuous non-decreasing function

(vii) $\lim_{s\to\infty} \Pi(\zeta, s) = 1$

(viii) $(\Xi(\zeta, s) = 0$ iff $\zeta = 0$

(ix) $\Xi(\lambda\zeta, s) = \Xi\left(\zeta, \frac{s}{|\lambda|}\right)$,

(x) $\Xi(\zeta, s) \diamond \Xi(y, s) \geq \Xi(\zeta + y, s + r)$,

(xi) $\Xi(\zeta, \diamond)$ is continuous non-increasing function,

(xii) $\lim_{s\to\infty} \Xi(\zeta, s) = 0$,

(xiii) $\Phi(\zeta, s) = 0$ iff $\zeta = 0$,

(xiv) $\Phi(\lambda\zeta, s) = \Phi\left(\zeta, \frac{s}{|\lambda|}\right)$,

(xv) $\Phi(\zeta, s) \diamond \Phi(y, s) \geq \Phi(\zeta + y, s + r)$,

(xvi) $\Phi(\zeta, \diamond)$ is continuous non-increasing function,

(xvii) $\lim_{s\to\infty} \Phi(\zeta, s) = 0$,

(xviii) If $s \leq 0$, then $\Pi(\zeta, s) = 0, \Xi(\zeta, s) = 1, \Phi(\zeta, s) = 1$.

In such case, $X = (\Pi, \Xi, \Phi)$ is called a neutrosophic normed (NN).

Example 2.1. Suppose $(\mathcal{Y}, \|\cdot\|)$ be a normed space, where $\|y\| = |y|, \forall y \in \mathbb{R}$. Give the function as $\zeta \circ y = \zeta + y - \zeta y$ and define $\zeta \star y = \min(\zeta, y)$, For $s > \|y\|$,

$$\Pi(\zeta, s) = \frac{s}{s + \|\zeta\|}, \Xi(\zeta, s) = \frac{\zeta}{s + \|\zeta\|}, \Phi(\zeta, s) = \frac{\|\zeta\|}{s} \tag{2.2}$$

$\forall \zeta, y \in X$ and $s > 0$. If we take $s \leq \|\zeta\|$, then

$\Pi(\zeta, s) = 0, \Xi(\zeta, s) = 1$ and $\Phi(\zeta, s) = 1$

Hence, $(\mathcal{Y}, X, \circ, \star)$ is neutrosophic norm space such that $X: X \times R^+ \to [0,1]$.

Definition 2.5. Let $(\mathcal{Y}, X, \star, \diamond)$ be a NNS. A sequence $x = \{x_n\}$ in $\mathcal{Y}$ is said to be convergent to α_1 with regard to NN- $X \Leftrightarrow$ for each $\psi > 0, \kappa \in (0,1), \exists N \in \mathbb{N}$such that

$$\Pi(x_n - \alpha_1, \psi) > 1 - \kappa, \Xi(x_n - \alpha_1, \psi) < \kappa, \Phi(x_n - \alpha_1, \psi) < \kappa, \forall n \in N$$

i.e, $\psi > 0$, we have

$$\lim_{n\to\infty} \Pi(x_n - \alpha_1, \psi) = 1, \lim_{n\to\infty} \Xi(x_n - \alpha_1, \psi) = 0 \text{ and } \lim_{n\to\infty} \Phi(x_n - \alpha_1, \psi) = 0.$$

We specify $X - \lim x_n = \alpha_1$.

Theorem 2.1. Let $(\mathcal{Y}, X, \star, \diamond)$ be a NNS. Then, a sequence $x = \{x_n\}$ in $\mathcal{Y}$ is convergent to $\alpha \in X$ if and only if $\lim_{n\to\infty} \Pi(x_n - \alpha, \psi) = 1, \lim_{n\to\infty} \Xi(x_n - \alpha, \psi) = 0$ and $\lim_{n\to\infty} \Phi(x_n - \alpha, \psi) = 0$.

Definition 2.6. [27,28] Assemblage of subsets $I \subseteq 2^{\mathbb{N}}$ is known as an ideal in $\mathbb{N}$ if I satisfies these condition;

(1) $\emptyset \in I$

(2) $\mathcal{H}, \mathcal{K} \in I \Rightarrow \mathcal{H} \cup \mathcal{K} \in I$, (additive);

(3) $\mathcal{H} \in I, \mathcal{K} \subseteq \mathcal{H} \Rightarrow \mathcal{K} \in I$. (hereditary);

If $I \neq 2^{\mathbb{N}}$, then $I \subseteq 2^{\mathbb{N}}$ is called nontrivial. A nontrivial ideal $I \subseteq 2^{\mathbb{N}}$ is said to be admissible if I includes every singleton subset of $\mathbb{N}$.

If there isn't a non-trivial ideal $K \neq I$, then I is the maximum non-trivial ideal such that $I \subset K$.

Definition 2.7. [27,28] Assemblage of subsets $F \subseteq 2^{\mathbb{N}}$ is known as a filter in $\mathbb{N}$ if I satisfies these condition:

(1) $\emptyset \notin F$,

(2) For $\mathcal{H}, \mathcal{K} \in F \Longrightarrow \mathcal{H} \cap \mathcal{K} \in F$,

(3) If $\mathcal{H} \in F$ and $\mathcal{K} \supset \mathcal{H}$ implies $\mathcal{K} \in F$.

The following definitions are given in [29].

Definition 2.8. Suppose $\{x_n\}$is a sequence in $(\mathcal{Y}, X,\star,\diamond)$. A sequence $\{x_n\}$ is said to be ideally convergent to α with regard to NN-X, if, for every $\epsilon > 0$ and $\psi > 0$

$$(P = \{n \in \mathbb{N}: \mathcal{U}(x\_n - \alpha, \psi) \leq 1 - \epsilon\text{" } or \text{ "} \mathcal{V}(x\_n - \alpha, \psi) \geq \epsilon, \mathcal{W}(x\_n - \alpha, \psi) \geq \epsilon\} \in I.)$$

It is denoted by $I_X - \lim x_n = \alpha$ or $x_n \to \alpha$.

Definition 2.9. Suppose $\{x_n\}$ be a sequence in $(\mathcal{Y}, X,\star,\diamond)$. A sequence $\{x_n\}$ is said to be an ideally Cauchy sequence with regard to $\mathrm{NN} - X$, if, for every $\kappa > 0$ and $\psi > 0, \exists k \in \mathbb{N}$ s.t

$$Q = \{n \in \mathbb{N}: \Pi(x_n - x_k, \psi) \leq 1 - \kappa \text{ or } \Xi(x_n - x_k, \psi) \geq \kappa, \Phi(x_n - x_k, \psi) \geq \kappa\} \in I.$$

3. RESULTS AND DISCUSSION

This study explores different topological features of neutrosophic ideal statistical convergent difference sequence spaces defined by the modulus function, which is a variant of ideal convergent sequence spaces. Let ω denote the space of all real sequences.

$$I_{st(\Delta)}^{0(X)}(f) = \Big\{z = \{z_{qs}\} \in \omega: \Big\{(q,s) \in \mathbb{N}^2 : \frac{1}{qs}\big|\{q \leq m, s \leq n: f\big(\Pi(\Delta z_{mn}, \psi)\big) \leq 1 - \kappa \text{ or } f\big(\Xi(\Delta z_{mn}, \psi)\big) \geq \kappa, f\big(\Phi(\Delta z_{mn}, \psi)\big) \geq \kappa\}\big| \geq \rho\Big\} \in I_2\Big\},$$

$$I_{st(\Delta)}^{(X)}(f) = \Big\{z = \{z_{qs}\} \in \omega : \Big\{(q,s) \in \mathbb{N}^2 \text{:for some } \psi \in \mathbb{R}, \frac{1}{qs}\big|\{q \leq m, s \leq n: f\big(\Pi(\Delta z_{mn} - \beta, \psi)\big) \leq 1 - \kappa \text{ or } f\big(\Xi(\Delta z_{mn} - \beta, \psi)\big) \geq \kappa, f\big(\Phi(\Delta z_{mn} - \beta, \psi)\big) \geq \kappa\}\big| \geq \rho\Big\} \in I_2\Big\}.$$

An open ball with radius $\kappa \in (0,1), \rho > 0$ and a center zAn open ball with radius $\psi > 0$, is denoted by $B(z, \kappa, \rho, \psi)$and is described as follows:

$$B(z, \kappa, \rho, \psi) = \Big\{t = \{t_{mn}\} \in \omega : \Big\{(q,s) \in \mathbb{N}^2 \text{:for some } \psi \in \mathbb{R}, \frac{1}{qs}\big|\{q \leq m, s \leq n: f\big(\Pi(\Delta z_{mn} - \Delta t_{mn}, \psi)\big) \leq 1 - \kappa \text{ or } f\big(\Xi(\Delta z_{mn} - \Delta t_{mn}, \psi)\big) \geq \kappa, f\big(\Phi(\Delta z_{mn} - \Delta t_{mn}, \psi)\big) \geq \kappa\}\big| \geq \rho\Big\} \in I_2\Big\}.$$

Theorem 3.1. The inclusion relation $I_{\mathrm{st}(\Delta)}^{0(X)}(f) \subset I_{\mathrm{st}(\Delta)}^{(X)}(f)$ holds.

The inverse of an inclusion relation does not hold. To back up our point, consider the instances below.

Example 3.1. Assume$(\mathbb{R}, \|.\|)$ be a normed spaces such that$\|z\| = \sup_{qs}|z_{qs}|$, and $z_1 * z_2 = \min\{z_1, z_2\}$ and $z_1 \diamond z_2 = \max\{z_1, z_2\}, \forall z_1, z_2 \in (0,1)$. For $\alpha > \|z\|$, now define norms $X = (\Pi, \Xi, \Phi)$ on $\mathbb{R}^2 \times (0, \infty)$ as follows:

$$\Pi(z, \alpha) = \frac{\alpha}{\alpha + \|z\|}, \Xi(z, \alpha) = \frac{\|z\|}{\alpha + \|z\|} \text{ and } \Phi(z, \alpha) = \frac{\|z\|}{\beta}$$

Then $(\mathbb{R}, X,\star,\diamond)$ is a NNS. Consider the sequence $(z_{mn}) = \{1\}$. It is easy to observe that $(z_{mn}) \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$ and $z_{mn} \overset{I_{\mathrm{st}(X)}}{\to} 1$, but $(z_{mn}) \notin I_{\mathrm{st}(\Delta)}^{0(X)}(f)$.

Lemma 3.1. Let $z = \{z_{mn}\} \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$. Then, $\forall \kappa \in (0,1)$, $\rho > 0$ and $\psi > 0$, the following assertions are equivalent,

(1) $I_{st(\Delta)}^{(X)}(f) - \lim z = \beta$,

(2) $\Big\{(q,s) \in \mathbb{N}^2$:for some $\psi \in \mathbb{R}, \frac{1}{qs}\big|\{q \leq m, s \leq n: f\big(\Pi(\Delta z_{mn} - \beta, \psi)\big) \leq 1 - \kappa$ or $f\big(\Xi(\Delta z_{mn} - \beta, \psi)\big) \geq \kappa, f\big(\Phi(\Delta z_{mn} - \beta, \psi)\big) \geq \kappa\}\big| \geq \rho\Big\} \in I_2$,

(3) $\Big\{(q,s) \in \mathbb{N}^2$:for some $\psi \in \mathbb{R}, \frac{1}{qs}\big|\{q \leq m, s \leq n: f\big(\Pi(\Delta z_{mn} - \beta, \psi)\big) > 1 - \kappa$ and $f\big(\Xi(\Delta z_{mn} - \beta, \psi)\big) < \kappa, f\big(\Phi(\Delta z_{mn} - \beta, \psi)\big) < \kappa\}\big| < \rho\Big\} \in \mathcal{F}(I_2)$,

(4) $I_2(st) - \lim f\left(\Pi\big(\Delta z_{qs} - \beta, \psi\big)\right) = 1, I_2(st) - \lim f\left(\Xi\big(\Delta z_{qs} - \beta, \psi\big)\right) = 0, I_2(st) - \lim f\left(\Phi\big(\Delta z_{qs} - \beta, \psi\big)\right) = 0.$

Theorem 3.2. The spaces $I_{\mathrm{st}(\Delta)}^{(X)}(f)$ and $I_{\mathrm{st}(\Delta)}^{0(X)}(f)$ are linear spaces.

Proof. We know that $I_{\mathrm{st}(\Delta)}^{0(X)}(f) \subset I_{\mathrm{st}(\Delta)}^{(X)}(f)$. Then, we show the outcome for $I_{\mathrm{st}(\Delta)}^{(X)}(f)$. The proof of linearity of the space $I_{\mathrm{st}(\Delta)}^{0(X)}(f)$ follows similarly.

Let $\{z_{mn}\}, \{t_{mn}\} \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$ and β_1, β_2 be scalars. The proof is trivial for $\beta_1 = 0$ and $\beta_2 = 0$. Now we take $\beta_1 \neq 0$ and $\beta_2 \neq 0$. For a given $\kappa > 0$, take $r > 0$ such that$(1 - \kappa) * (1 - \kappa) > (1 - r)$ and $\kappa \diamond \kappa < r$.

$$P_1 = \left\{(q,s) \in \mathbb{N}^2 : \frac{1}{qs}\left|\left\{q \leq m, s \leq n : f\left(\Pi\left(\Delta z_{mn} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) \leq 1 - \kappa \text{ or } f\left(\Xi\left(\Delta z_{mn} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) \geq \kappa, f\left(\Phi\left(\Delta z_{mn} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) \geq \kappa\right\}\right| \geq \rho\right\} \in I_2,$$

$$P_2 = \left\{(q,s) \in \mathbb{N}^2 : \frac{1}{qs}\left|\left\{q \leq m, s \leq n : f\left(\Pi\left(\Delta z_{mn} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) \leq 1 - \kappa \text{ or } f\left(\Xi\left(\Delta z_{mn} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) \geq \kappa, f\left(\Phi\left(\Delta z_{mn} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) \geq \kappa\right\}\right| \geq \rho\right\} \in I_2.$$

Now, we take the complement of P_1 and P_2

$$P_1^c = \left\{(q,s) \in \mathbb{N}^2 : \frac{1}{qs}\left|\left\{q \leq m, s \leq n : f\left(\Pi\left(\Delta z_{mn} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) > 1 - \kappa \text{ and } f\left(\Xi\left(\Delta z_{mn} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) < \kappa, f\left(\Phi\left(\Delta z_{mn} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) < \kappa\right\}\right| < \rho\right\} \in F(I_2),$$

$$P_2^c = \left\{(q,s) \in \mathbb{N}^2 : \frac{1}{qs}\left|\left\{q \leq m, s \leq n : f\left(\Pi\left(\Delta z_{mn} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) > 1 - \kappa \text{ and } f\left(\Xi\left(\Delta z_{mn} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) < \kappa, f\left(\Phi\left(\Delta z_{mn} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) < \kappa\right\}\right| < \rho\right\} \in F(I_2).$$

Consequently, set $P = \mathcal{P}_1 \cup \mathcal{P}_2$ produces $P \in I_2$. Thus, P^c is a set that is not empty in $\mathcal{F}(I_2)$. We'll illustrate this for each $\{z_{qs}\}, \{t_{qs}\} \in I^{(X)}_{\mathrm{st}(\Delta)}(f)$.

$$P^c \subset \left\{(q,s) \in \mathbb{N}^2 : \frac{1}{qs}\left|\{q \leq m, s \leq n : f\big(\Pi(\varsigma\Delta z_{mn} + \omega\Delta t_{mn}) - (\varsigma\beta_1 + \omega\beta_2, \psi)\big) > 1 - \kappa \text{ and } f\big(\Xi(\varsigma\Delta z_{mn} + \omega\Delta t_{mn}) - (\varsigma\beta_1 + \omega\beta_2, \psi)\big) < \kappa, f\big(\Phi(\varsigma\Delta z_{mn} + \omega\Delta t_{mn}) - (\varsigma\beta_1 + \omega\beta_2, \psi)\big) < \kappa\}\right| < \rho\right\}.$$

Let $(i,j) \in \mathcal{P}^c$. In this case,

$$f\left(\Pi\left(\Delta z_{ij} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) > 1 - \kappa \text{ and } f\left(\Xi\left(\Delta z_{ij} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) < \kappa, f\left(\Phi\left(\Delta z_{ij} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) < \kappa,$$

$$f\left(\Pi\left(\Delta t_{ij} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) > 1 - \kappa \text{ and } f\left(\Xi\left(\Delta t_{ij} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) < \kappa, f\left(\Phi\left(\Delta t_{ij} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) < \kappa.$$

Consider

$$\begin{aligned} f\left(\Pi(\varsigma\Delta z_{ij} + \omega\Delta t_{ij}) - (\varsigma\beta_1 + \omega\beta_2, \psi)\right) &\geq f\left(\Pi\left(\varsigma\Delta z_{ij} - \varsigma\beta_1, \frac{\psi}{2}\right)\right) \star f\left(\Pi\left(\omega\Delta t_{ij} - \omega\beta_2, \frac{\psi}{2}\right)\right) \\ &= f\left(\Pi\left(\Delta z_{ij} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) \star f\left(\Pi\left(\Delta t_{ij} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) \\ &> (1-\kappa) * (1-\kappa) > 1 - r, \end{aligned}$$

$$\Rightarrow f\left(\Pi(\varsigma\Delta z_{ij} + \omega\Delta t_{ij}) - (\varsigma\beta_1 + \omega\beta_2, \psi)\right) > 1 - r.$$

$$\begin{aligned} f\left(\Xi(\varsigma\Delta z_{ij} + \omega\Delta t_{ij}) - (\varsigma\beta_1 + \omega\beta_2, \psi)\right) &< f\left(\Xi\left(\varsigma\Delta z_{ij} - \varsigma\beta_1, \frac{\psi}{2}\right)\right) \diamond f\left(\Xi\left(\omega\Delta t_{ij} - \omega\beta_2, \frac{\psi}{2}\right)\right) \\ &= f\left(\Xi\left(\Delta z_{ij} - \beta_1, \frac{\psi}{2|\varsigma|}\right)\right) \diamond f\left(\Xi\left(\Delta t_{ij} - \beta_2, \frac{\psi}{2|\omega|}\right)\right) < \kappa \diamond \kappa < r, \end{aligned}$$

$$\Rightarrow f\left(\Xi(\varsigma\Delta z_{ij}+\omega\Delta t_{ij})-(\varsigma\beta_1+\omega\beta_2,\psi)\right)<r$$

and

$$\begin{aligned}f\left(\Xi(\varsigma\Delta z_{ij}+\omega\Delta t_{ij})-(\varsigma\beta_1+\omega\beta_2,\psi)\right)&<f\left(\Phi\left(\varsigma\Delta z_{ij}-\varsigma\beta_1,\frac{\psi}{2}\right)\right)\\&\diamond f\left(\Phi\left(\omega\Delta t_{ij}-\omega\beta_2,\frac{\psi}{2}\right)\right)\\&=f\left(\Phi\left(\Delta z_{ij}-\beta_1,\frac{\psi}{2|\varsigma|}\right)\right)\\&\diamond f\left(\Phi\left(\Delta t_{ij}-\beta_2,\frac{\psi}{2|\omega|}\right)\right)<\kappa\diamond\kappa<r,\end{aligned}$$

$$\Rightarrow f\left(\Phi(\varsigma\Delta z_{ij}+\omega\Delta t_{ij})-(\varsigma\beta_1+\omega\beta_2,\psi)\right)<r.$$

Thus

$$\begin{aligned}P^c\subset\Big\{(q,s)\in\mathbb{N}^2:\frac{1}{qs}\big|\{q\le m,s&\le n: f\big(\Pi(\varsigma\Delta z_{mn}+\omega\Delta t_{mn})-(\varsigma\beta_1+\omega\beta_2,\psi)\big)>1-r \text{ and } f\big(\Xi(\varsigma\Delta z_{mn}+\omega\Delta t_{mn})-(\varsigma\beta_1+\omega\beta_2,\psi)\big)\\&<r, f\big(\Phi(\varsigma\Delta z_{mn}+\omega\Delta t_{mn})-(\varsigma\beta_1+\omega\beta_2,\psi)\big)<r\}\big|<\rho\Big\}.\end{aligned}$$

Since $P^c\in\mathcal{F}(I_2)$, thus by the properties of $\mathcal{F}(I_2)$ we have

$$\begin{aligned}\Big\{(q,s)\in\mathbb{N}^2:\frac{1}{qs}\big|\{q\le m,s&\le n: f\big(\Pi(\varsigma\Delta z_{mn}+\omega\Delta t_{mn})-(\varsigma\beta_1+\omega\beta_2,\psi)\big)>1-r \text{ and } f\big(\Xi(\varsigma\Delta z_{mn}+\omega\Delta t_{mn})-(\varsigma\beta_1+\omega\beta_2,\psi)\big)\\&<r, f\big(\Phi(\varsigma\Delta z_{mn}+\omega\Delta t_{mn})-(\varsigma\beta_1+\omega\beta_2,\psi)\big)<r\}\big|<\rho\Big\}\in\mathcal{F}(I_2).\end{aligned}$$

Hence $I_{\mathrm{st}(\Delta)}^{(X)}(f)$ is a linear space.

Theorem 3.3. Every closed ball $B^c(z,\kappa,\rho,\psi)$ is an open in $I_{\mathrm{st}(\Delta)}^{(\Pi,\Xi,\Phi)}(f)$, where neutrosophic parameter $\psi,\rho>0$ with center at z and radius $0<\kappa<1$.

Proof. Assume that $B(z,\rho,\psi,\kappa)$ is an open ball with a radius of $0<\kappa<1$ and a neutrosophic parameter $\psi,\rho>0$, with its centre at $z=(z_{mn})\in I_{\mathrm{st}(\Delta)}^{(X)}(f)$.

$$\begin{aligned}B(z,\kappa,\rho,\psi)(f)=\Big\{t&\in I_{\mathrm{st}(\Delta)}^{(X)}(f):\frac{1}{qs}\big|\{q\le m,s\le n: f\big(\Pi(\Delta z-\Delta t,\psi)\big)\le 1-\kappa \text{ or } f\big(\Xi(\Delta z-\Delta t,\psi)\big)\\&\ge\kappa, f\big(\Phi(\Delta z-\Delta t,\psi)\big)\ge\kappa\}\big|\ge\rho\Big\}\in I_2.\end{aligned}$$

Then

$$\begin{aligned}B^c(z,\kappa,\rho,\psi)(f)=\Big\{t&\in I_{\mathrm{st}(\Delta)}^{(X)}(f):\frac{1}{qs}\big|\{q\le m,s\le n: f\big(\Pi(\Delta z-\Delta t,\psi)\big)>1-\kappa \text{ and } f\big(\Xi(\Delta z-\Delta t,\psi)\big)\\&<\kappa, f\big(\Phi(\Delta z-\Delta t,\psi)\big)<\kappa\}\big|<\rho\Big\}\in\mathcal{F}(I_2).\end{aligned}$$

suppose $t\in\mathcal{B}^c(z,\kappa,\rho,\psi)$. Then, for

$$f\big(\Pi(\Delta z-\Delta t,\psi)\big)>1-\kappa \text{ and } f\big(\Xi(\Delta z-\Delta t,\psi)\big)<\kappa, f\big(\Phi(\Delta z-\Delta t,\psi)\big)<\kappa,$$

so there exists $\psi_0\in(0,\gamma)$ such that $f\big(\Pi(\Delta z-\Delta t,\psi_0)\big)>1-\kappa$ and $f\big(\Xi(\Delta z-\Delta t,\psi_0)\big)<\kappa, f\big(\Phi(\Delta z-\Delta t,\psi_0)\big)<\kappa$.

Let $\kappa_0=f\big(\Pi(\Delta z-\Delta t,\psi_0)\big)$, we have $\kappa_0>1-\kappa$. Then $\exists u\in(0,1)$ such that $\kappa_0>1-u>1-\kappa$. For $\kappa_0>1-u$, we can have $\kappa_1,\kappa_2,\kappa_3\in(0,1)$ such that $\kappa_0*\kappa_1>1-u,(1-\kappa_0)\diamond(1-\kappa_2)<u$. and $(1-\kappa_0)\diamond(1-\kappa_3)<u$.

Let $\kappa_4=\max\{\kappa_1,\kappa_2,\kappa_3\}$. Then $(1-u)<\kappa_0*\kappa_1\le\kappa_0*\kappa_4$ and $(1-\kappa_0)\diamond(1-\kappa_4)\le(1-\kappa_0)\diamond(1-\kappa_2)<u$. Consider the closed ball $B^c(t,\psi-\psi_0,\rho,1-\kappa_4)$ and $B^c(z,\kappa,\rho,\psi)$.

We prove that $B^c(t,\psi-\psi_0,\rho,1-\kappa_4)\subset B^c(z,\kappa,\rho,\psi)$. Let $v=\{v_{mn}\}\in\mathcal{B}^c(t,\psi-\psi_0,\rho,1-\kappa_4)$. Then $f\big(\Pi(\Delta t-\Delta v,\psi,\psi-\psi_0)\big)>\kappa_4$ and $f\big(\Xi(\Delta t-\Delta v,\psi,\psi-\psi_0)\big)<1-\kappa_4, f\big(\Phi(\Delta t-\Delta v,\psi,\psi-\psi_0)\big)<1-\kappa_4$. Therefore

$$\begin{aligned}f\big(\Pi(\Delta z-\Delta t,\psi)\big)&\ge f\big(\Pi(\Delta z-\Delta t,\psi_0)\big)\star f\big(\Pi(\Delta t-\Delta v,\psi,\psi-\psi_0)\big)\\&\ge\kappa_0\star\kappa_4\ge\kappa_0\star\kappa_1\\&>(1-u)>(1-\kappa)\\f\big(\Xi(\Delta z-\Delta t,\psi)\big)&\le f\big(\Xi(\Delta z-\Delta t,\psi_0)\big)\diamond f\big(\Xi(\Delta t-\Delta v,\psi,\psi-\psi_0)\big)\\&\le(1-\kappa_0)\diamond(1-\kappa_4)\le\kappa_0\diamond\kappa_2\\&<u<\kappa\end{aligned}$$

and

$$\begin{aligned}f(\Phi(\Delta z-\Delta t,\psi))&\le f\big(\Phi(\Delta z-\Delta t,\psi_0)\big)\diamond f\big(\Phi(\Delta t-\Delta v,\psi,\psi-\psi_0)\big)\\&\le\kappa_0\diamond\kappa_4\le\kappa_0\diamond\kappa_3\\&<u<\kappa.\end{aligned}$$

Therefore, the set

$$\left\{t \in I_{\mathrm{st}(\Delta)}^{(X)}(f) : \frac{1}{qs}\left|\{q \leq m, s \leq n : f\big(\Pi(\Delta z - \Delta t, \psi)\big) > 1-\kappa \text{ and } f\big(\Xi(\Delta z - \Delta t, \psi)\big) < \kappa, f\big(\Phi(\Delta z - \Delta t, \psi)\big) < \kappa\}\right| < \rho\right\} \in \mathcal{F}(I_2).$$

$\Longrightarrow v = (v_{mn}) \in B^c(z, \kappa, \rho, \psi) \Longrightarrow B^c(t, \psi - \psi_0, \rho, 1 - \kappa_4) \subset B^c(z, \kappa, \rho, \psi)$.

Remark 3.2. It is clear that $I_{\mathrm{st}(\Delta)}^{(X)}(f)$ is a neutrosophic normed space with respect to neutrosophic norms $X = (\Pi, \Xi, \Phi)$. Define

Now define a collection $\tau_{\mathrm{st}(\Delta)}^{(X)}(f)$ of a subset of $I_{\mathrm{st}(\Delta)}^{(X)}(f)$ as follows: $\tau_{\mathrm{st}(\Delta)}^{(X)}(f) = \{P \subset I_{\mathrm{st}(\Delta)}^{(X)}(f)$: for every $z = (z_{mn}) \in P, \exists \psi > 0$ and $\kappa \in (0,1)$, $\rho > 0$ such that $\mathcal{B}^c(z, \kappa, \rho, \psi) \subset P\}$. Then $\tau_{\mathrm{st}(\Delta)}^{(X)}(f)$ is a topology on $I_{\mathrm{st}(\Delta)}^{(X)}(f)$.

Theorem 3.4. The topology $\tau_{\mathrm{st}(\Delta)}^{(X)}(f)$ on the space $I_{\mathrm{st}(\Delta)}^{(X)}(f)$ is first countable.

Proof. For every $z = (z_{mn}) \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$, suppose the set $\mathcal{B} = \left\{B^c\left(z, \frac{1}{mn}, \frac{1}{mn}\right)\right\} : m, n = 1,2,3,4, \ldots\}$, which is a local countable basis at $z \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$. As a result, the topology $\tau_{\mathrm{st}(\Delta)}^{(X)}(f)$ on the space $I_{\mathrm{st}(\Delta)}^{0(X)}(f)$ is first countable.

Theorem 3.5. The spaces $I_{\mathrm{st}(\Delta)}^{(X)}(f)$ and $I_{\mathrm{st}(\Delta)}^{0(X)}(f)$ are Hausdorff spaces.

Proof. We know that $I_{\mathrm{st}(\Delta)}^{0(X)}(f) \subset I_{\mathrm{st}(\Delta)}^{(X)}(f)$. We will only show the solution for $I_{\mathrm{st}(\Delta)}^{(X)}(f)$. Let $z = (z_{mn}), y = (y_{mn}) \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$ such that $z \neq y$. Then, $0 < f(\Pi(\Delta z - \Delta y, \psi)) < 1, 0 < f(\Xi(\Delta z - \Delta y, \psi)) < 1$ and $0 < f(\Phi(\Delta z - \Delta y, \psi)) < 1$.Putting $\kappa_1 = f(\Pi(\Delta z - \Delta y, \psi)), \kappa_2 = f(\Xi(\Delta z - \Delta y, \psi)), \kappa_3 = f(\Phi(\Delta z - \Delta y, \psi))$ and $\kappa = \max\{\kappa_1, 1-\kappa_2, 1-\kappa_3\}$. Then, for each $\kappa_0 \in (\kappa, 1)$ there exist $\kappa_4, \kappa_5, \kappa_6 \in (0,1)$ such that $\kappa_4 * \kappa_4 \geq \kappa_0, (1-\kappa_5) \diamond (1-\kappa_5) \leq (1-\kappa_0)$ and $(1-\kappa_6) \diamond (1-\kappa_6) \leq (1-\kappa_0)$.

Once again putting $\kappa_7 = \max\{\kappa_4, 1-\kappa_5, 1-\kappa_6,\}$, think about the closed balls. $B^c\left(z, 1-\kappa_7, \frac{\psi}{2}\right)$ and $B^c\left(y, 1-\kappa_7, \frac{\psi}{2}\right)$ respectively centered at z and y. Then, it is obvious that $B^c\left(z, 1-\kappa_7, \frac{\psi}{2}\right) \cap \mathcal{B}^c\left(y, 1-\kappa_7, \frac{\psi}{2}\right) = \phi$.If possible let $z = \{z_{mn}\} \in \mathcal{B}^c\left(z, 1-\kappa_7, \frac{\psi}{2}\right) \cap \mathcal{B}^c\left(y, 1-\kappa_7, \frac{\psi}{2}\right)$. Then we have,

$$\begin{aligned}
\kappa_1 &= f(\Pi(\Delta z - \Delta y, \psi)) > \kappa_7 \star \kappa_7 \geq \kappa_4 \star \kappa_4 \geq \kappa_0 > \kappa_1 \\
\kappa_2 &= f(\Xi(\Delta z - \Delta y, \psi)) < (1-\kappa_7) \diamond (1-\kappa_7) \leq (1-\kappa_5) \diamond (1-\kappa_5) \leq (1-\kappa_0) < \kappa_2
\end{aligned} \tag{3.1}$$

and

$$\kappa_3 = f(\Phi(\Delta z - \Delta y, \psi)) < (1-\kappa_7) \diamond (1-\kappa_7) \leq (1-\kappa_6) \diamond (1-\kappa_6) \leq (1-\kappa_0) < \kappa_3. \tag{3.2}$$

We have a contradiction from equations (3.1) and (3.2). Therefore, $\mathcal{B}^c\left(z, 1-\kappa_7, \frac{\psi}{2}\right) \cap \mathcal{B}^c\left(y, 1-\kappa_7, \frac{\psi}{2}\right) = \phi$. Hence the space $I_{\mathrm{st}(\Delta)}^{0(X)}(f)$ is a Hausdorff space.

Theorem 3.6. Let $z = \{z_{mn}\} \in \omega$ be a sequence. If $\exists$ a sequence $y = \{y_{mn}\} \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$ such that $f\big(\Delta(z_{mn})\big) = f\big(\Delta(y_{mn})\big)$ for almost all m, n relative to neutrosophicI_2, then $z \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$.

Proof. Consider $f\big(\Delta(z_{mn})\big) = f\big(\Delta(y_{mn})\big)$ for almost all m, n relative to neutrosophic I_2. Then $\{(m,n) \in \mathbb{N}^2 : f\big(\Delta(z_{mn})\big) \neq f\big(\Delta(y_{mn})\big)\} \in I_2$. This implies. $\{(m,n) \in \mathbb{N}^2 : f\big(\Delta(z_{mn})\big) = f\big(\Delta(y_{mn})\big)\} \in \mathcal{F}(I_2)$. Therefore for $(m,n) \in \mathcal{F}(I_2), \forall \psi > 0$,

$$f\left(\Pi\left(\Delta z_{mn} - \Delta y_{mn}, \frac{\psi}{2}\right)\right) = 1, f\left(\Xi\left(\Delta z_{mn} - \Delta y_{mn}, \frac{\psi}{2}\right)\right) = 0 \text{ and } f\left(\Phi\left(\Delta z_{mn} - \Delta y_{mn}, \frac{\psi}{2}\right)\right) = 0.$$

Since $\{y_{mn}\} \in I_{\mathrm{st}(\Delta)}^{(X)}(f)$, let (y_n) is $\Delta(I_2)$-convergent to α. Then for any $\kappa \in (0,1)$ and $\psi > 0$,

$$A_1 = \left\{(q,s) \in \mathbb{N}^2 : \frac{1}{qs}\left|\left\{q \leq m, s \leq n : f\left(\Pi\left(\Delta y_{mn} - \alpha, \frac{\psi}{2}\right)\right) > 1-\kappa \text{ and } f\left(\Xi\left(\Delta y_{mn} - \alpha, \frac{\psi}{2}\right)\right) < \kappa, f\left(\Phi\left(\Delta y_{mn} - \alpha, \frac{\psi}{2}\right)\right) < \kappa\right\}\right| < \rho\right\} \in \mathcal{F}(I_2),$$

Consider the set,

$$A_2 = \left\{(q,s) \in \mathbb{N}^2 : \frac{1}{qs}\left|\left\{q \leq m, s \leq n : f\left(\Pi\left(\Delta z_{mn} - \alpha, \frac{\psi}{2}\right)\right) > 1-\kappa \text{ and } f\left(\Xi\left(\Delta z_{mn} - \alpha, \frac{\psi}{2}\right)\right) < \kappa, f\left(\Phi\left(\Delta z_{mn} - \alpha, \frac{\psi}{2}\right)\right) < \kappa\right\}\right| < \rho\right\}.$$

We show that $A_1 \subset A_2$. So for $(m,n) \in A_1$ we have,

$$f(\Pi(\Delta z_{mn}-\alpha,\psi)) \geq f\left(\Pi\left(\Delta z_{mn}-\Delta y_{mn},\frac{\psi}{2}\right)\right) \star f\left(\Pi\left(\Delta y_{mn}-\alpha,\frac{\psi}{2}\right)\right) > 1 \star (1-\kappa) = 1-\kappa$$

$$f(\Xi(\Delta z_{mn}-\alpha,\psi)) \leq f\left(\Xi\left(\Delta z_{mn}-\Delta y_{mn},\frac{\psi}{2}\right)\right) \diamond f\left(\Xi\left(\Delta y_{mn}-\alpha,\frac{\psi}{2}\right)\right) < 0 \diamond \kappa = \kappa$$

and

$$f(\Phi(\Delta z_{mn}-\alpha,\psi)) \leq f\left(\Phi\left(\Delta z_{mn}-\Delta y_{mn},\frac{\psi}{2}\right)\right) \diamond f\left(\Phi\left(\Delta y_{mn}-\alpha,\frac{\psi}{2}\right)\right) < 0 \diamond \kappa = \kappa.$$

$\Rightarrow (m,n) \in A_2$ and hence $A_1 \subset A_2$. Since $A_1 \in \mathcal{F}(I_2)$, therefore $A_2 \in \mathcal{F}(I_2)$. Hence $z = \{z_{mn}\} \in I^{(X)}_{\mathrm{st}(\Delta)}(f)$.

4. CONCLUSIONS

In this study, we broaden the concept of I_2-statistical convergence within neutrosophic normed spaces by incorporating difference sequences and modulus functions. We also define the new concept of I_2-statistically convergent difference sequences in neutrosophic normed spaces through modulus functions and examine some of their fundamental properties.

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