Published by SET Publisher

Journal of Basic & Applied Sciences

ISSN (online): 1927-5129

On the Distribution of the Prime Numbers

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Article Info:

Keywords:
Number theory,
Even numbers,
Prime numbers,
Complex numbers,
Prime number distribution.

Timeline:
Received: October 19, 2024
Accepted: November 17 2024
Published: December 12, 2024

Citation: Khalil LA. On the distribution of the prime numbers. J Basic Appl Sci 2024; 20: 182-189.

DOI: https://doi.org/10.29169/1927-5129.2024.20.17

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Abstract:

This research explores the distribution of prime numbers, which are a fundamental topic in number theory. The study originated from the author's fascination with mathematics and the desire to discover something novel. The research proposes that the distribution of prime numbers follows a regular pattern starting from the number 2. The author suggests that prime numbers can be obtained by dividing certain even numbers that have four factors by the number 2, resulting in prime numbers in sequential order. This hypothesis was tested and confirmed through the practical application of the proposed mathematical formula. Additionally, the study found that even numbers greater than or equal to 8, with six or more factors, produce complex numbers. Thus, this research provides two main contributions: firstly, a mathematical formula for the distribution of prime numbers, and secondly, a formula for the distribution of complex numbers. These findings have potential applications in various mathematical fields, including cryptography and problem-solving in number theory.

1. INTRODUCTION

Numbers are one of the most intriguing topics in mathematics. Since ancient times, they have fascinated and astonished scientists, as well as amateurs like myself, who are passionate about mathematics. Prime numbers such as 2, 3, 5, 7, 11, 13, 17, and so on, appear to be randomly distributed within the set of natural numbers, yet they exhibit certain regularities. These numbers are considered the fundamental building blocks for other numbers, similar to the role of atoms in relation to molecules. They have important applications in cryptography and data protection, and are central to the Riemann Hypothesis, one of the most famous mathematical problems of our time [1,2].

Over the course of thousands of years, mathematicians have uncovered only a fraction of the mysteries surrounding prime numbers. Euclid, the Greek mathematician, demonstrated that there are infinitely many prime numbers. The basic idea is that if there were a finite number of prime numbers, by multiplying them together and adding one, one could produce a new number that is not divisible by any of the prime numbers in the list, thus proving there must be infinitely many prime numbers [3].

In the nineteenth century, mathematicians formulated the prime number theorem, which provides an approximate estimate of the number of prime numbers less than a given number. The theorem reveals that prime numbers become less frequent as numbers grow larger, following a specific approximate mathematical formula [4]. The study of twin primes, pairs of prime numbers with exactly one number between them, posits that there are infinitely many such pairs. Despite the conjecture's plausibility, it has neither been proven nor disproven. In a breakthrough in 2013, mathematician Yitang Zhang demonstrated that there are infinitely many pairs of prime numbers with gaps smaller than 70 million. This was a significant milestone in narrowing the constraints on prime number gaps [5].

Goldbach's conjecture, another prominent study, suggests that every even number greater than 2 can be expressed as the sum of two prime numbers. While this conjecture holds for small numbers and has been verified for numbers up to 4 quintillion using modern computers and sophisticated software, it has yet to be proven for all even numbers [4]. The investigation of palindromic primes, which read the same forwards and backwards, reveals that such primes are rare among reversible numbers regardless of the numerical system used. A notable example is Belphegor's Prime: 1000000000000066600000000000 001 [3].

The Riemann Hypothesis, one of the Millennium Prize Problems, extends the theory of prime numbers by providing a more precise formula that estimates the number of prime numbers less than a given number. This hypothesis, which links the zeros of a special complex function to the distribution of prime numbers, remains unproven despite substantial numerical evidence supporting it [2,6].

1.1. Literature Review

Over the course of a thousand years, mathematicians have discovered only a little about prime numbers. Some of them are summarized as follows:

Firstly, Greek mathematician Euclid (330 - 275 BC) showed that there are an infinite number of prime numbers. The basic idea is that if there is a finite number of prime numbers and a list of that prime numbers, we can add one to their product, thus producing a new number that is not divisible by any of the prime numbers in that list. That numbers will either be a prime number that is not in our list, or it will have a prime divisor that is not in our list. In both cases, this contradicts the idea that prime numbers are finite, and therefore there must be an infinite number of them.

Secondly, in the nineteenth century, mathematicians proved some theorems concering prime numbers, whch allow information with regard to bigger prime numbers. The theory gives an approximate estimate of the number of prime numbers smaller than the given number, and prime numbers become rarer among large numbers according to a certain approximate mathematical formula.

Finally, initial twin study such that a prime twin is a pair of prime numbers with one number between them.

Example 1: For example, 5,7, 11 and 13, 29 and 31 are twin primes. Indeed, twin primes states that there are an infinite number of twin prime pairs among the infinite number of prime numbers.

We should state that most mathematicians believe that this conjecture must be true, since while prime numbers become rarer as numbers become increasingly larger, mathematicians' theoretical experience and intuition with prime numbers suggest that prime twin pairs should appear from time to time.

However, the conjecture has neither been proven nor disproved.

After remaining an open question in every sense of the word for centuries, in 2013, Yitang Zhang, who is a Chinese-American mathematician and primarily working on number theory at the University of California, Santa Barbara, made a surprising breakthrough in solving the problem, earning him a Genius MacArthur Fellowship in September 2014. Indeed, Yitang Zhang established a new method that showed that there is an infinite number of twin prime pairs, and the number of numbers between them exceeded seventy million, a huge number, but it represents the first specific restriction on the distances between prime numbers that was discovered.

Besides, in the fall of 2013, a large group of mathematicians dealt with Zhang's work, and obtained similar results and collaboratively found smaller constraints, ultimately proving that there are many pairs of prime numbers with at most 246 of them.

Furtermore, Goldbach proved that any even number greater than the number 2 can be written as the sum of two prime numbers. This result certainly applies to small numbers. However, the conjecture was not proven for all even numbers.

Example 2: 4 = 2 + 2, 8 = 5 + 3, 20 = 13 + 7.

In 21st century, using improved computers and well-designed software, researchers have been able to verify the above conjecture for even numbers up to 4 quintillion (4.000.000.000.000.000.000). This is very good evidence for the validity of the conjecture. However, in mathematics, this is not enough to say that the conjecture is true for all numbers. If it was true for all numbers smaller than a certain limit or a certain number, then even if it is unreasonably large.

As for studying reciprocal prime numbers, the number of hearts is a number or word that can be read in the same form even if the order of the numbers or letters is reversed, for example, the word radar or the number 191.Likewise, reversible primes are prime numbers that would read the same if their order was reversed.

One of the special reversible primes is Belphegor's Prime:

1.000.000.000.000.066.600.000.000.000.001,

which is the number 1 followed by 13 zeros, then 666, then another 13 zeros, and then ending with 1.

As with the prime-twin conjecture, it is currently unknown whether there are an infinite number of reversible or symmetrical primes, however, research activity regarding the latter conjecture is less active than for the prime-twin.

Problems such as the Goldbach conjecture and the prime twin depend only on the structure and distribution of the primes themselves.

However, the intuition of reversible prime numbers depends on the number system that is used. The reversible numbers in the binary system are completely different from those in the decimal system. For example, the prime number 31 in binary is written as 11111. This number is hearts in binary, but not hearts in decimal. While mathematicians study reciprocal primes, they have concluded that such primes are rare among reciprocal numbers regardless of the number system used, in number theory more effort is devoted to problems that focus mostly on properties of prime numbers independent of their representation.

1.2. Study of the Riemann Hypothesis

The Riemann hypothesis is one of the millennium prize problems, a collection of the most important unsolved problems in mathematics, for which the solution of one of these problems is accompanied by a prize of one million US dollars.

The Riemann hypothesis is an extension of the theory of prime numbers mentioned above. This theory gives a mathematical formula for the approximate number of prime numbers smaller than a certain large number. Indeed, the Riemann hypothesis gives a more specific result, as it provides a formula that shows how accurate the result of the previous theory's formula is.

The great 19th century mathematician Bernhard Riemann linked the constraints of this precision to a special function at the level of complex numbers. The Riemann hypothesis states that all points on the plane of complex numbers lie on a certain line in the plane when that function equals zero. If so, then the precision constraints would also be true. As with the other problems on this list, there is a fair amount of numerical evidence for the Riemann hypothesis, and most mathematicians believe it to be true.

Mathematicians have tested billions of points equal to zero in this function and found that all points lie on this line.

As with all the other problems we have looked at, there is still no complete proof of the hypothesis. In all of these cases, while most mathematicians believe these conjectures to be true, and there is little experimental evidence for them, the search for complete proof continues.

1.3. Method

This research adopts a mathematical approach to explore the arrangement and properties of prime numbers. A prime number is defined as an integer greater than one that has no divisors other than one and itself. The initial prime numbers in the numerical series are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. In contrast, composite numbers are those with more than two natural factors, excluding the number one, which is neither prime nor composite.

2. IDENTIFICATION OF PRIME NUMBERS

To ascertain the primality of numbers, two primary methods are employed:

2.1. Factoring

Through factorization, mathematicians can swiftly determine a number's primality. A factor is any number that, when multiplied by another, yields the original number. For instance, the prime factors of 10 are 2 and 5, as their product is 10. However, 1 and 10 are not considered prime factors.

2.2. Utilization of Calculators

Calculators aid in the division process to test for primality. For example, dividing 57 by 2 yields a non-integer result, while dividing by 3 gives 19, an integer, indicating that 57 is not a prime number as it has factors other than one and itself.

3. HISTORICAL CONTEXT OF PRIME NUMBERS

Prime numbers have been studied since antiquity, with Greek mathematicians like Euclid and Eratosthenes contributing significantly to their understanding. Euclid provided the first proof of the infinitude of prime numbers. The distribution of prime numbers is further elucidated by the Prime Number Theorem and the Riemann Zeta Function.

3.1. Prime Numbers in Cryptography

The RSA encryption system exemplifies the application of prime numbers in computing. It requires two large prime numbers for secure information transmission over the Internet. The difficulty of prime factorization underpins the security of RSA encryption [7].

3.2. Properties of Prime Numbers

The research will investigate several properties of prime numbers, such as [8, 9]:

- Every prime number greater than 3 can be expressed in the form $6k \pm 1$ where k is a natural number.
- Every integer greater than 1 has at least one prime divisor.
- If n is composite, it has a prime divisor p less than or equal to $\sqrt{n}$.
- Prime twins are pairs of prime numbers with a difference of 2, such as $(5, 7)$ and $(11, 13)$.

3.3. Research Hypotheses

The research hypotheses will be tested using a methodological approach that includes:

- Detailed analysis of prime number distribution.
- Application of cryptographic principles to demonstrate the practical utility of prime numbers.
- Exploration of prime number properties through computational methods [8, 9].

4. RESULTS AND DISCUSSION

In this results and discussion chapter, we outline empirical findings that support the hypothesis that the distribution of prime numbers can be determined through division of even numbers that have exactly four factors. Through the factor tree method, we showed that even numbers with four factors, when divided by two, yield the known prime numbers. In contrast, even numbers with more than four factors yield composite numbers. This finding not only strengthens our understanding of the nature of prime numbers but also offers a new perspective in the search for larger primes, which has important implications in the fields of cryptography and number theory.

4.1. Empirical Analysis of Prime Number Distribution

The study’s empirical analysis began with the hypothesis that prime numbers can be derived from even numbers starting from 4, provided they have

exactly four factors. The function for even numbers is defined as [10-12]:

$$F(n) = 2n$$

Applying this function, we observe the following sequence:

$$F(1) = 2, F(2) = 4, F(3) = 6, ..$$

For odd numbers, the function is:

$$F(n) = 2n + 1$$

Yielding the sequence:

$$F(1) = 3, F(2) = 5, F(3) = 7, F(1) = 3, F(2) = 5, F(3) = 7, \ldots..$$

The hypothesis posits that prime numbers are the result of dividing even numbers (with exactly four factors) by 2. The equation used is [13-15]:

$$P = \frac{e}{2}, \text{ where } e \geq 4 \text{ and } e \text{ has 4 factors}$$

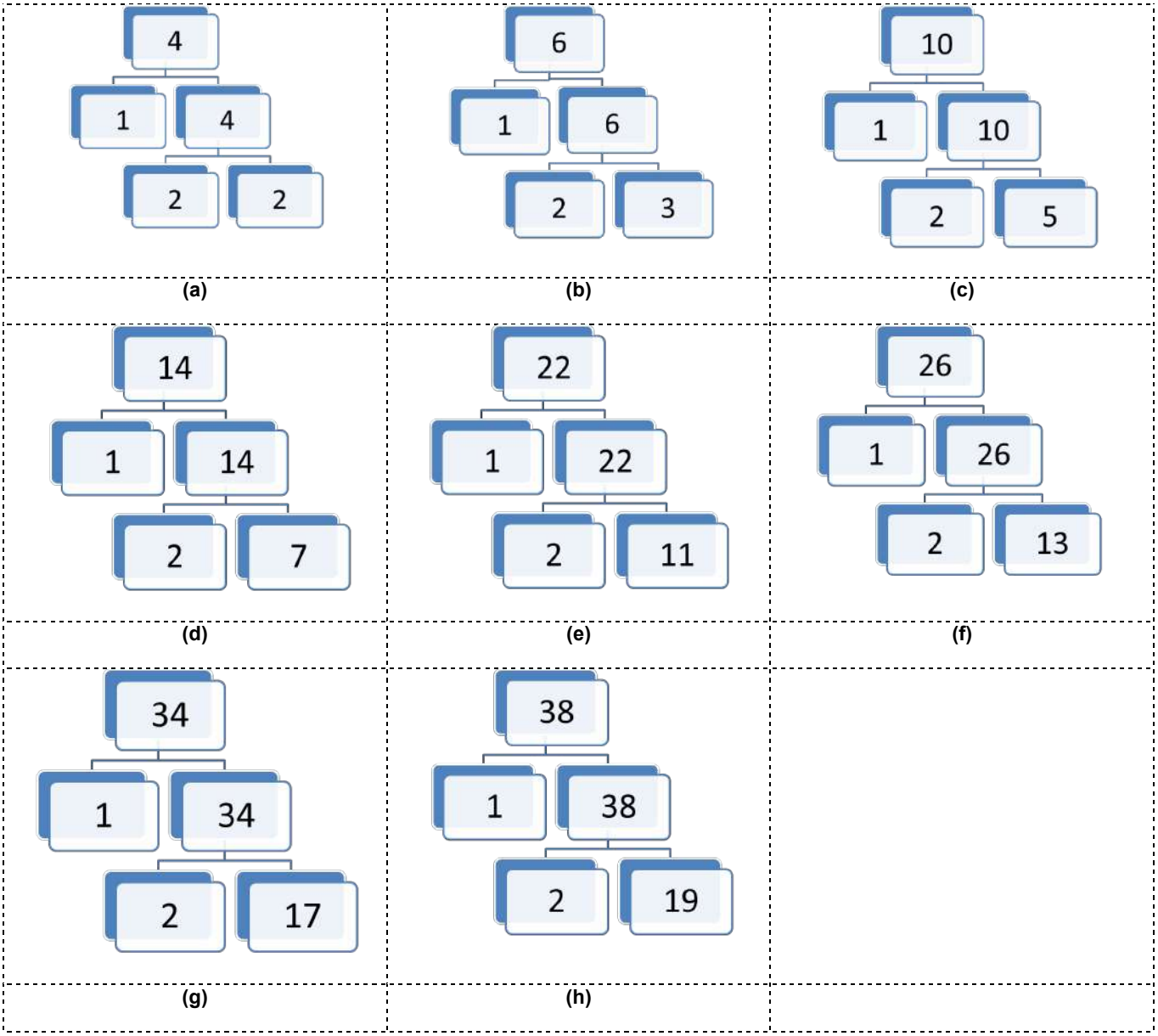

Figure 1: The first figure (**a**), the second figure (**b**), the third figure (**c**), the fourth figure (**d**), the fifth figure (**e**), the sixth figure (**f**), the seventh figure (**g**), and the eighth figure (**h**).

The following results were obtained:

$$\frac{4}{2} = 2, \frac{6}{2} = 3, \frac{10}{2} = 5, \frac{14}{2} = 7, \frac{22}{2} = 11, \frac{26}{2} = 13, \frac{34}{2} = 17, \frac{38}{2} = 19,$$

These results align with the known sequence of prime numbers.

Figures **1**, illustrate the known sequence of prime numbers. Each figure depicts the next prime number in the established sequence of prime numbers. This data provides a clear visual representation of how prime numbers evolve and are distributed among larger numbers [16, 17].

4.2 Factor Tree Method Illustration

The factor tree method was employed to illustrate the factors of the numbers in question. The method visually represents the factorization process, breaking down a composite number into its prime factors. The illustration (created using the graphic_art tool) demonstrates the factorization of even numbers with exactly four factors [18, 19].

4.3. Testing the Hypothesis on Composite Numbers

To further validate the hypothesis, even numbers equal to or greater than 8 with six or more factors were divided by 2. The equation used is [20]:

$c = \frac{e}{2}$, where $e \geq 8$ and e has 6 factors or more

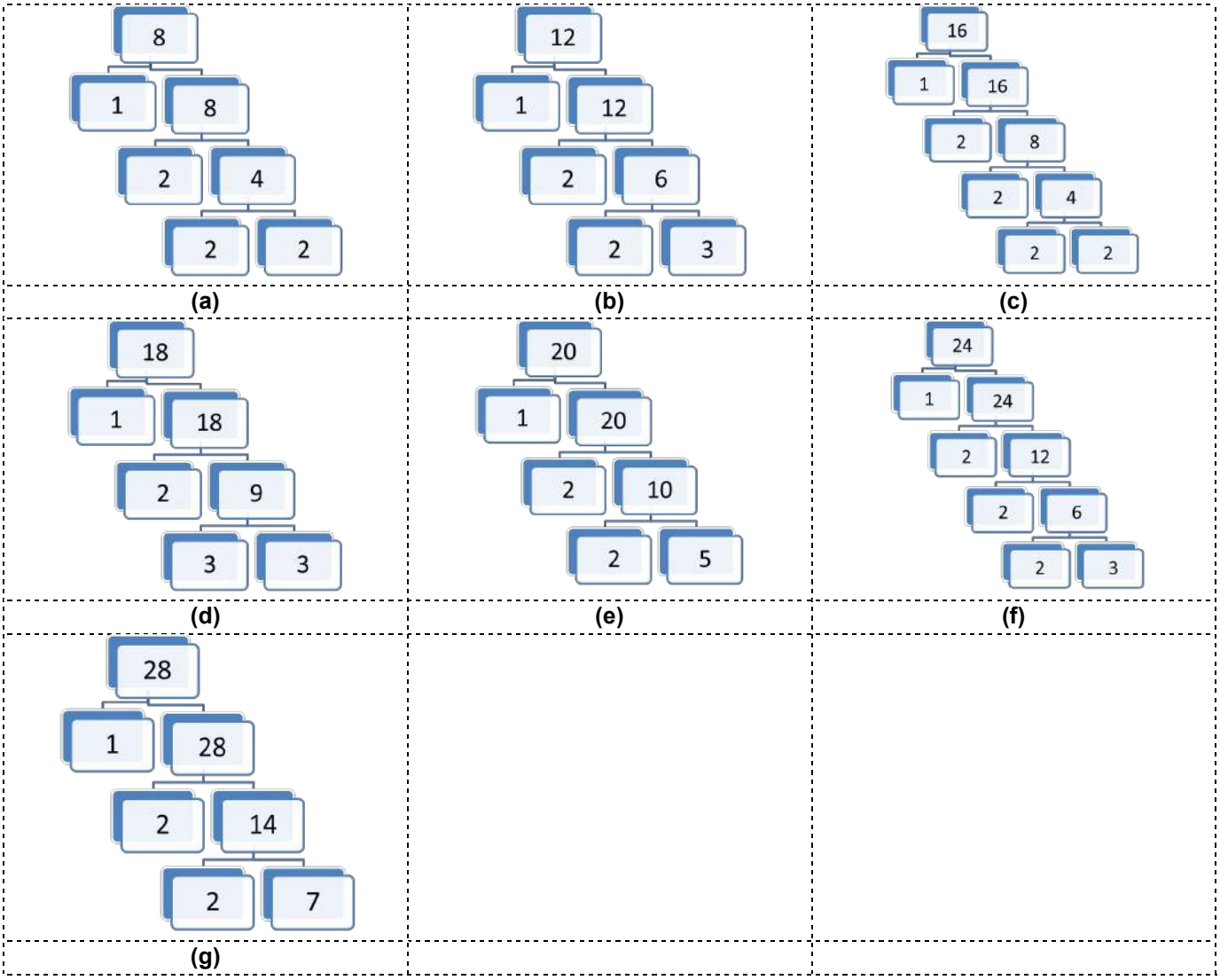

Figure 2: The first figure (**a**), the second figure (**b**), the third figure (**c**), the fourth figure (**d**), the fifth figure (**e**), the sixth figure (**f**), the seventh figure (**g**).

The results were as follows:

$$\frac{8}{2}=4,\frac{12}{2}=6,\frac{16}{2}=8,\frac{18}{2}=9,\frac{20}{2}=10,\frac{24}{2}=12,\frac{28}{2}=14.$$

These results consistently yielded composite numbers, supporting the hypothesis that the distribution of composite numbers follows a sequential pattern.

Figure **2**, depicted results consistently generated composite numbers, reinforcing the hypothesis that composite numbers follow a sequential pattern in their distribution. Each illustration showcases a series of composite numbers arranged in a sequential manner, indicating a discernible trend in their distribution within the numerical spectrum. This observation contributes to a deeper understanding of the underlying structure and distribution of composite numbers within the realm of mathematics.

5. DISCUSSION

The findings suggest a novel approach to identifying prime numbers through the division of even numbers with specific factors. This method offers a systematic way to explore the distribution of primes and could potentially simplify the search for larger prime numbers. The factor tree method provides a clear visual representation of the factors, aiding in the understanding of the number's composition.

The study's results contribute to the broader field of number theory by proposing a practical mathematical approach to prime number identification. Further research is needed to explore the implications of this method and its potential applications in cryptography and other areas of mathematics.

6. CONCLUSION

Through this research, I reached the conclusion that the distribution of prime numbers is regular according to the theoretical and practical aspects, and that is that the distribution of prime numbers is a regular order in ascending order starting from the number 2 and continuing to infinity of prime numbers. It is a quotient of the division of (the even numbers starting from the number 4, provided that It has four factors) on the number 2, and the validity of this hypothesis was confirmed according to a regular and proven equation I arrived at another conclusion, which is that the rest of the even numbers that are equal to the number 8 and greater than it and that have factors of 6 or more produce complex numbers for us, meaning that here we also obtained the distribution of the complex numbers in ascending order, starting from the number 4 and so on to infinity of complex numbers. The order of these numbers is actually regular and according to a regular and proven equation

ACKNOWLEDGEMENT

Author could express a gratitude here. For reference list, we encourage author to prioritize looking for sources from journal first.

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