Published by SET Publisher

Journal of Basic & Applied Sciences

ISSN (online): 1927-5129

Further Operations (Complement, Intersection, Union) for IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set and their Applications

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Article Info:

Keywords:
HyperSoft Set;
IndetermSoft Set;
IndetermSoft Algebra;
IndetermHyperSoft Set;
IndetermSoft Operators.

Timeline:
Received: December 26, 2024
Accepted: January 18, 2025
Published: February 13, 2025

Citation: Adesina AS. Further Operations (Complement, Intersection, Union) for IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set and their Applications. J Basic Appl Sci 2025; 21: 47-52.

DOI: https://doi.org/10.29169/1927-5129.2025.21.06

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Abstract:

In this paper, efforts are intensified as much as possible to explicitly and clearly give the definitions as regards the operations involving the complement, intersection as well as the union for IndetermSoft Set, IndetermHyperSoft Set, and Tree

Soft Set respectively. All these we believe, to the best of our knowledge, have neither been pronounced nor treated in literatures, and to use them in real applications.

AMS Mathematics Subject Classification 2020: Primary: 03E20, 03E72, 03E75Secondary:03E65.

1. INTRODUCTION

In this paper, our aim is to define the operations (complement, intersection, union) for IndetermSoft Set and IndetermHyperSoft Set respectively and give explicit real life examples where the concepts is applicable.

1.1. Some Basic Operational Definitions

1.1.1. Determinate and Indeterminate Soft Operators

Subsequently, the following operators are going to be applicable henceforth. They have originally been defined by Florentin [3-5].

(i). joinAND

joinAND, or put together, denoted by ⩕, defined as:

x ⩕ y = x and y, or put together x and y; herein the conjunction “and” has the common sense from the natural language.

(ii). Disjoin OR (see [3])

DisjoinOR, or separate in parts, denoted by ⩔, defined as:

x disjoinOR y = x ⩔ y = $\{x\}$, or $\{y\}$, or both $\{x, y\}$

= x, or y, or both x and y; (see [3])

(iii). Exclusive OR

exclusiveOR, meaning either one, or the other; it is an IndetermSoft Operator (to choose among two alternatives).

h1 ⩔E h2 = either h1, or h2, and no both {h1, h2}. (see [3])

(iv). NOT

NOT, or no, or sub-negation/sub-complement, denoted by ⫬, where

$\mathrm{NOT}(h) =$ ⫬ $h = no\ h$, in other words all elements from H, except h, either single elements, or two elements, …, or $n - 1$ elements from $H - \{h\}$, or the empty element ∅. (see [3])

2. INDETERMSOFT SET

Definition (see [3])

Suppose that is the universe of discourse, H a non-empty subset of U, and P(H) the powerset of H. Let a be an attribute, and A be a set of this attribute values.

A function $F\colon A \to (H)$ is called an IndetermSoft Set (or Function) if:

i. the set A has some indeterminacy;

ii. or P(H) has some indeterminacy;

iii. or there exist at least an attribute value v ∈A, such that F(v) = indeterminate (unclear, uncertain, or not unique);

iv. or any two or all three of the above situations.

Definition: (please, see [3] and [4])

Let U be a universe of discourse, H a non-empty subset of U, and H(⩕, ⩔, ⩔E, ⫬) the IndetermSoft Algebra generated by closing the set H under the operators ⩕, ⩔, ⩔E, and ⫬.

Let a be an attribute, with its set of attribute values denoted by A. Then the pair (F, A), where $F\colon A \to$ H(⩕,⩔, ⩔E, ⫬), is called an IndetermSoft Set over H.

2.1. Complement

Let U be a universe of discourse, H a non-empty subset of U, and P(H) the powerset of H. Let x be an attribute, and A, a set of this attribute-values.

Definition (please see ([9] and [10]))

The complement of a soft set (F, A)can be denoted by $(F, A)^c$.This can be defined by $(F, A)^c = (F^c, \urcorner A)$. Here, $F^c{:}\urcorner A \to P(H)$ is a mapping. This mapping is given by $F^c(\boldsymbol{\theta}) = U - F(\urcorner\boldsymbol{\theta})$ for all $\boldsymbol{\theta}$ in ⌉A. It should be well noted here that every complement of any given set is another set in its own right. Hence, from this definition, one can infer that $(F, A)^c = (F^c, \urcorner A)$ is the complement of the IndetermSoft Set (F, A). (This is how the complement in this particular context is being defined).

Now, just like the properties of the normal indetermSoft set, it is expedient we investigate the validity of the following axioms also for the complement of the indetermSoft set:

(i) The set ⌉A must have certain indeterminacy;

(ii) or the set P(H) has some indeterminacy;

(iii) or there exist at least an attribute-value u ∈⌉A, such that F^c(u) is indeterminate (unclear, incomplete, conflicting, or not unique);

(iv) or any two or all three of the above situations.

Our next line of action here is to check whether the complement of every indetermSoftset would in the real sense satisfy those given conditions. This, we are going to do using some real life examples and situations for our justification.

2.2. Real Example of the complement of IndetermSoft Set:

For simplicity sake, allowU, to be the universe of discourse which represents the set of some selected students in a given higher school, given by : U = {a, b, c, d, e, f, g, h}. Suppose that three students with the highest performances are represented by H, a non-empty subset of U.

Let this be given by: H = {b, d, e, f, g}, and assuming that these students are to be considered for the attribute A of size such that A = {tall, short}. If these attributes are further expressed as :

A = {t,s}. Now, given that membership functions for the attribute set A to be A = {(t, [0.7, 0.8](t)T, [0.3,0.4](t)I, [0.1,0.2](t)F), (s, [0.8,0.9](s)T, [0.4,0.5](s)I, [0.2,0.3](s)F)}. Note here also that our membership valued functions are expressed in the form of ranges. (These are typical examples of where the concepts of the indetermSoftsets as well as the indeterminacy are applicable).

Now, here comes the Indeterminacy with respect to the function.

i) If peradventure somebody should ask a source:

- Which student(s) is/are tall among the students with the highest performance ?

Answer:

- By my observation, only one student seems to be very tall amidst them. I am not sure which student that is, but I think the student should either be e or b.

Hence, we have that F(t) = b or e (This is an indeterminate / uncertain answer).

This means for example, that $F_1(t)$ is approximately equivalent to $F_1(t)$ = b(0.8, 0.3, 0.1), (This is to say that the chance that b is tall is 80% true, indeterminate-chance of tall-not tall is 30%, and chance that bis not tall is 10%) or that $F_1(t)$ is approximately equivalent to $F_1(t)$ = e(0.9, 0.4 0.2), (This is to say that the chance that e is tall is 90% true, indeterminate-chance of tall-not tall is 40%, and chance that bis not tall is 20%) (In most cases, our real world is full of a lot of uncertainties, vagueness and a lot of lack of full assurances).

We have that: $\urcorner F_1(t) = F_1(\urcorner t)$ = b(t, (1 - 0.8)(t)F, (1 - 0.3)(t)I, (1 - 0.1)(t)T)

= b(t, (0.2)(t)F, (0.7)(t)I, (0.9)(t)T)

ii) If the question is asked again:

- But, which of the students are short ?

Answer: - I can not say, the only thing I know is that student e is not short. This is because I know him very well. In fact, he is my friend.

Therefore, F(s) = not e (again indeterminate / uncertain answer).

So, if $F_2(s)$ = e is approximately equivalent to $F_2(s)$ = e(0.8(s)T, 0.4(s)I, 0.3(s)F), or the chance that e is short is 80% true, indeterminate-chance of short – not short is 40%, and chance that $h_2$not short is 30%. Then, $F_2(s)$ = not e = ˥e = ˥e((1 - 0.8)F, (1 - 0.4)I, (1 - 0.3)T) = ˥e(0.2F, 0.6I, 0.7T), and $\urcorner F_2(s) = F_2(\urcorner s)$ = not (not e) = ˥(˥**e**) = e(0.8T, 0.4I, 0.3F), as required.

By definition, the complement of A can as well be calculated as follows:

˥A = {(t,(1 - [0.7, 0.8])(t)T, (1 - [0.3,0.4])(t)I, (1 - [0.1,0.2])(t)F), (s, (1 - [0.8,0.9])(s)T,(1 - [0.4,0.5])(s)I,(1 - [0.2,0.3])(s)F)}

= {(t,([0.2, 0.3])(t)T, ([0.6,0.7])(t)I, ([0.8,0.9])(t)F), (s, ([0.1,0.2])(s)T,([0.5,0.6])(s)I, ([0.7,0.8])(s)F)} (Our readers should kindly consult [6] for comprehensive details on calculations involving intervals. In [6] the arithmetic involving interval values were dealt with rudimentally in the concepts of the neutrosophic statistics).

Thus, ˥**F**(t) can either be in H or ˥H or the null set. Similarly for˥$\mathbf{F}_2$(s).

From here, it can clearly be inferred that the aforementioned axioms are fully satisfied. Hence, the following proposition is very immediate.

Proposition: Every indetermSoft set possesses an indetermSoft complement set.

2.3. Intersection

Definition (please see [10]). The intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set given by: (H, C), where C = A∩B, and for all e in C, H(e) = F(e) = G(e). Hence, this could be denoted by : (F, A) $\tilde{\cap}$ (G, B) = (H, C)

Now, using the example given before, let B = {a,c,e}. Then, (F, A) $\tilde{\cap}$ (G, B) = (H, C), where C = {a,e}. By observation, it is very clear that the intersection of any two indetermSoftset is also an indetermSoft set. This is simply because all our four axioms in question would be satisfied as required.

2.4. Proposition

Let U be a universe of discourse, H a non-empty subset of U, and P(H) the power set of H. Let x_1 and $x_2$be attributes of sets A and B respectively such that A, B $\subset U$. Then, the following statements are equivalent:

(i). If (F, A) or (G, B) or both are indetermSoft sets, then so is (H, C) = (F, A)$\tilde{\cap}$(G, B)

(ii). If(F, A)$\tilde{\cap}$G, B) = (H, C), happens to be an indetermSoft set, then (F, A) or (G, B) or both of them are indetermSoft sets.

2.5. Union

Definition (please see [9] and [10]). The union of two soft sets (F,A) and (G, B) over a common universe U is the soft set (H, C), where C = A∪B, and for all e in C,

$$H(e) = \begin{cases} F(e), \text{if } e \in A\backslash B, \\ G(e), \text{if } e \in B\backslash A, \\ F(e) \cup G(e), \text{if } e \in A \cap B \end{cases}$$

The union can also be denoted by: (F,A)$\tilde{\cup}$(G,B) = (H,C), where. C = A∪B.

If the same example is followed, we are going to have from here that, C = A∪B = {a,b,c,e}, and by observation, the four axioms for the indetermSoft set are satisfied. Hence, th following assertion surfaces.

Proposition:

The union of two indetermSoft set is also an indetermSoft set. We also have as follows :

Proposition:

Let U be a universe of discourse, H a non-empty subset of U, and P(H) the power set of H. Let x_1 and $x_2$be attributes of sets A and B respectively such that A, B $\subset U$. Then, the following statements are equivalent :

(i). If (F, A) or (G, B) or both are indetermSoft sets, then so is (H, C) = (F, A)$\tilde{\cup}$(G, B)

(ii). If (F, A)$\tilde{\cup}$(G, B) = (H, C), happens to be an indetermSoft set, then (F, A) or (G, B) or both of them are indetermSoft sets.

3. INDETERMHYPERSOFT SET

Definition:

Let U be a universe of discourse, H a non-empty subset of U, and P(H) the power set of H. Let a_1, a_2, …, a_n, where $n \geq 1$, be n distinct attributes, whose corresponding attribute-values are respectively the sets A_1, A_2, …, A_n, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in$ {1, 2, …, n}.

Then the pair (F, $A_1 \times A_2 \times \ldots \times A_n$), where $A_1 \times A_2 \times \ldots \times$ represents the Cartesian product, with F: $A_1 \times A_2 \times \ldots \times A_n \rightarrow (H)$ is called an IndetermHyperSoft Set if:

(i) at least one of the sets A_1, A_2, …, A_n has some indeterminacy;

(ii) or the set P(H) has some indeterminacy;

(iii) or there exist at least one n-plet (e_1, e_2,..., e_n) $\in A_1 \times A_2 \times \ldots \times A_n$ such that F(e_1, e_2,..., e_n) = indeterminate (unclear, uncertain, conflicting, or not unique);

iv) or any two or all three of the above situations. (Please, see [4] and [8])

Definition: (see [8])

Let U be a universe of discourse, H a non-empty subset of U, and H(⩕, ⩖, ⩖E, ⫬) the IndetermSoft Algebra generated by closing the set H under the operators ⩕, ⩖, ⩖E, and ⫬.

Let $a1$, $a2$, … an, where $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A1$, $A2$, …, An, with $Ai \cap Aj = \emptyset$ for $i \neq j$, and i, $j \in$ {1, 2, …, n}. Then the pair (F, $A1 \times A2 \times \ldots \times An$), where $A1 \times A2 \times \ldots \times An$ represents a Cartesian product, with

F: $A1 \times A2 \times \ldots \times An \rightarrow$ H(⩕, ⩖, ⩖E, ⫬), is called an IndetermHyperSoft Set.

Similarly, one may associate fuzzy / intuitionistic fuzzy / neutrosophic etc. degrees and extend the IndetermHyperSoft Set to some Fuzzy / Intuitionistic Fuzzy / Neutrosophic etc. IndetermHyperSoft Set.

3.1. Complement

Now, considering the description of the IndetermHyperSoft Set as given above, we have the following definition:

Definition (please see ([9] and [10])). The complement of a soft set (F, A) can be denoted by $(F, A)^c$. This can be defined by $(F, A)^c = (F^c, \urcorner A)$. Here, $F^c : \urcorner A \rightarrow P(H)$ is a mapping. This mapping is given by $F^c(\theta) = U - F(\urcorner\theta)$ for all θ in $\urcorner A$. It should be well noted here that every complement of any given set is another set in its own right. Hence, from this, setting $A = A_1 \times A_2 \times ... \times A_n$ and $e = (e_1, e_2, ..., e_n)$, one can infer that : $(F, A)^c = (F^c, \urcorner A)$ is the complement of the IndetermHyperSoft Set (F, A). And by observations, the complement of every IndetermHyperSoft Set is always an IndetermHyperSoft Set.

3.2. Intersection

By the definition of intersection, setting $A = A_1 \times A_2 \times ... \times A_n$ and $e = (e_1, e_2, ..., e_n$, it could be inferred that the intersection of two or more IndetermHyperSoft Set is always another set which is also an IndetermHyperSoft Set. (see [3] and [4])

3.3. Union

By setting $A = A_1 \times A_2 \times ... \times A_n$ we conclude that the union of two or more IndetermHyperSoft Set is always another set which is also an IndetermHyperSoft Set. (kindly see [3] for more details).

4. TREESOFT SET

Definition (please, see [3] and [4]):

Suppose that U is the universe of discourse, and H a non-empty subset of U, with P(H) the power set of H and suppose that A is the set of attributes (parameters, factors, etc.), $A = \{A_1, A_2, ... A_n\}$ for integer $n \geq 1$

where $A_1, A_2, ... A_n$ are attributes of first level (since they have one-digit indexes). Each attribute A_i, $1 \leq i \leq n$, is formed by sub-attributes:

$A_1 = \{A_{11}, A_{12}, ...\}$, $A_2 = \{A_{21}, A_{22}, ...\}$,... $A_n = \{A_{n1}, A_{n2}, ...\}$,

where A_{ij} are sub-attributes (or attributes of second level) (since they have two-digit indexes).

Again, each sub-attribute A_{ij}, is formed by sub-sub-attributes (or attributes of third level): A_{ijk}

And so on, as much refinement as needed into each application, up to sub-sub-…-sub-attributes (or attributes of m-level (or having m digits into the indexes): $A_{i1,\ i2,\ ...\ in.}$ A graph-tree Tree (A), is formed, whose root is A (considered of level zero), then nodes of level 1, level 2, up to level m. (please, see [14]) We call leaves of the graph-tree, all terminal nodes (nodes that have no descendants). Then the TreeSoft Set is: $F :(P(Tree(A)) \rightarrow P(H)$.

Tree(A) is the set of all nodes and leaves (from level 1 to level m) of the graph-tree, and P(Tree(A)) is the powerset of the Tree(A).

Consequences

By implications, the following propositions are hereby observed:

4.1. Proposition: (Complement)

By the definitions, every complement of a TreeSoft set is another TreeSoft set.

4.2. Proposition: (Intersection)

The intersection of two or more TreeSoft sets is always a TreeSoft set

4.3. Proposition: (Union)

The union of two or more TreeSoft sets is a TreeSoft set.

Applications in General

Our real world is full of a lot of vagueness as well as so many uncertainties and complex situations and entities. Hence, the concepts of this studies go a long way in proffering relevant solutions to many of the problems of uncertainties from time to time.

FUTURE RESEARCH

We leave proofs involving 4.1 - 4.3 for further future research open problems.

ACKNOWLEDGEMENTS

The author expresses deep gratitude to the reviewers for their useful suggestions and corrections which have finally improved the quality of the work considerably.

AUTHOR STATEMENT (AUTHORS CONTRIBUTIONS)

The complete compilation, full arrangements and the final preparation was carried out by the first author Sunday Adesina Adebisi.

ETHICAL CONDUCT

The research was done in accordance with relevant laws, regulations and institutional consideration.

FUNDING

This research received no specific grant from any funding agency in the public, commercial, or any not-for-profit sectors whatsoever.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its supplementary Information.

CONFLICT OF INTEREST

The author declares that there is no competing or conflict of interests whatsoever.

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